IITB-Monash Research Academy

Indian Institute of Technology Bombay | MONASH University

An Indian-Australian research partnership

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| **Project Title:** | **Theoretical and numerical analysis of multiphysics problems in thin structures** | |
| **Project Number** | **IMURA1087** | |
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| **IITB Department**: | Mathematics | |

## Research Clusters:

## Research Themes:

| **Highlight which of the Academy's CLUSTERS this project will address?** *(Please nominate JUST **one**. For more information, see www.iitbmonash.org)* | | **Highlight which of the Academy's Theme(s) this project will address?** *(Feel free to nominate more than one. For more information, see www.iitbmonash.org)* | |
|---|---|---|---|
| 1 | *Material Science/Engineering (including Nano, Metallurgy)* | ***1*** | *Advanced computational engineering, simulation and manufacture* |
| 2 | *Energy, Green Chem, Chemistry, Catalysis, Reaction Eng* | 2 | *Infrastructure Engineering* |
| 3 | *Math, CFD, Modelling, Manufacturing* | 3 | *Clean Energy* |
| 4 | *CSE, IT, Optimisation, Data, Sensors, Systems, Signal Processing, Control* | 4 | *Water* |
| 5 | *Earth Sciences and Civil Engineering (Geo, Water, Climate)* | 5 | *Nanotechnology* |
| 6 | *Bio, Stem Cells, Bio Chem, Pharma, Food* | 6 | *Biotechnology and Stem Cell Research* |
| 7 | *Semi-Conductors, Optics, Photonics, Networks, Telecomm, Power Eng* | 7 | *Humanities and social sciences* |
| 8 | *HSS, Design, Management* | 8 | Design |

## The research problem

This PhD thesis project aims to address the existing gap in the understanding of mathematical properties of multi-physics systems, and their numerical approximation. The focus of the project is on the analysis of novel mathematical formulations and schemes for coupled systems in the mechanics of thin plates. Rather than solely examining single-physics systems, the project will delve into the theory and computation of interactions between solid deformation and various additional effects, such as thermal, electric, chemical, and fluid interactions. For example, in fully coupled thermoelastic systems both temperature and displacement fields are considered as primary variables. In these systems, the solid deformation and heat transfer are closely connected, and the analysis of the system requires taking into account the coupled nature of the equations. We will develop mixed formulations for these systems, analyse their mathematical properties, and develop and analyse numerical schemes for these formulations. In addition to the fully coupled thermoelastic systems, we will also investigate other types of coupled systems, such as poroelasticity, where the solid deformation is coupled with fluid flow, and thermo-poroelasticity, where the solid deformation, heat transfer, and fluid flow are all coupled. We will develop mixed and mixed-primal formulations for these systems, and will analyse their mathematical properties and numerical approximation.

The primary objectives of the project include: 1) examining the solvability and regularity of the underlying continuum-based models using fixed-point theorems in non-standard functional spaces, and 2) investigating the construction and analysis of reliable and robust numerical methods (taking for example hybrid discretisations, virtual elements, or mixed finite elements). The fundamental properties of these methods, including physical relevance, well-posedness, convergence, stability, accuracy, and reliability, will be rigorously addressed.

To undertake this project, knowledge of functional analysis and operator theory, as well as a background in nonlinear partial differential equations (PDEs) and numerical methods, is required.

## Project aims

1) Establishing the solvability of the coupled PDE-based models for the electromechanics of thin structures. This step involves advanced theoretical techniques including fixed-point theory, saddle-point and double saddle-point abstract problems, and non-standard regularity.
2) Studying new physics-preserving formulations for coupled multiphysics problems. Motivating examples come essentially from two applicative problems: developmental cell biology and cardiac electromechanics in the atria.
3) The third step focuses on proposing novel numerical schemes to solve the problems studied in points 1) and 2). We will address the rigorous analysis of important properties of the proposed methods, as well as their implementation and benchmarking.

## What is expected of the student when at IITB and when at Monash?

*Highlight how the project will gain from the students stay at IITB and at Monash*

At IITB:

- acquiring all necessary tools from the modern theory of PDEs
- being exposed to the theory of distributions, Sobolev space theory and variational approach to studying partial differential equations
- focus on qualitative study of solutions to nonlinear elliptic and parabolic problems
- command over various fixed point theorems and applications; A technical know-how of proving regularity results for elliptic and parabolic equations
- getting familiarity in the theory of mixed finite elements

At Monash:

- Mathematically modelling physical phenomena

- Formulating mathematical problems in continuum mechanics focusing on cardiac tissue and cell biology
- Obtaining basic notions in coding and scientific computing
- Developing the analysis of mixed methods

## Expected outcomes

- A novel framework for the rigorous mathematical analysis of a class of coupled systems of PDEs in shells and plates
- New aspects of continuum models for nonlinear diffusion-elasticity problems
- Robust and efficient numerical discretisations

## How will the project address the Goals of the above Themes?

The formalisms discussed above will have specific applications in areas such as skin growth, the macroscopic interaction between the deformation of atrial heart muscle and electrical activation, and the design and testing of smart materials, including actuators. This project has the potential to attract multiple collaborations, which will greatly benefit the IITB-Monash Research Academy. These applications are directly relevant to the goal of "advancing computational engineering, simulation, and manufacturing".

## Potential RPCs from IITB and Monash

Harsha Hutridurga- IITB Mathematics (closest to the topic apart from the IITB supervisor)
Santiago Badia - Monash Mathematics (closest to the topic apart from the Monash supervisors)

## Capabilities and Degrees Required

Degree: MSc in Mathematics

Strengths:
Functional analysis
Partial differential equations
Numerical analysis of PDEs
Scientific computing

## Necessary Courses

Numerical analysis
Partial differential equations
Continuum mechanics

## Potential Collaborators

Nikhil Medhekar, Monash Material Sciences / Amit Singh, IITB Mechanical Engineering. They are specialists on one applicable aspect of this project.

**Select up to (4) keywords from the Academy's approved keyword list (available at http://www.iitbmonash.org/becoming-a-research-supervisor/) relating to this project to make it easier for the students to apply.**

smart materials