IITB-Monash Research Academy

Indian Institute of Technology Bombay | MONASH University

An Indian-Australian research partnership

Project Title:	Robust Neural Network Approximation of Evolution Equations	
Project Number	**IMURA1263**	
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Research Clusters:

Research Themes:

Highlight which of the Academy's CLUSTERS this project will address? *(Please nominate JUST **one.** For more information, see www.iitbmonash.org)*		**Highlight which of the Academy's Theme(s) this project will address?** *(Feel free to nominate more than one. For more information, see www.iitbmonash.org)*	
1	*Material Science/Engineering (including Nano, Metallurgy)*	***1***	*Artificial Intelligence and Advanced Computational Modelling*
2	*Energy, Green Chem, Chemistry, Catalysis, Reaction Eng*	2	*Circular Economy*
3	*Math, CFD, Modelling, Manufacturing*	3	*Clean Energy*
4	*CSE, IT, Optimisation, Data, Sensors, Systems, Signal Processing, Control*	4	*Health Sciences*
5	*Earth Sciences and Civil Engineering (Geo, Water, Climate)*	5	*Smart Materials*
6	*Bio, Stem Cells, Bio Chem, Pharma, Food*	6	*Sustainable Societies*
7	*Semi-Conductors, Optics, Photonics, Networks, Telecomm, Power Eng*	7	*Infrastructure*

8	*HSS, Design, Management*		

The research problem

Solving Partial Differential Equations (PDEs) accurately and efficiently is central to scientific computing and mathematical modelling. In recent years, **Physics-Informed Neural Networks (PINNs)** have emerged as a data-driven approach for solving partial differential equations (PDEs) and have been gaining popularity [1]. Using neural networks, PINNs approximate the solution of a PDE by incorporating the differential equation and boundary conditions directly into the training loss, typically as discretisations of the residuals measured in the L^2 norm. The method offers flexibility and the potential for solving high-dimensional and inverse problems. However, **PINNs suffer from several known limitations**, including unstable training, sensitivity to hyperparameters, and lack of robustness in the presence of stiff or multiscale phenomena.

To improve the mathematical structure and stability of PINNs, **Variational Physics-Informed Neural Networks (VPINNs)** have been introduced [2]. VPINNs cast the PDE in its variational (or weak) form, replacing pointwise residual minimisation with minimisation of residuals tested against a basis of functions. This formulation brings the method closer to classical Finite Element methods-based approaches. Nevertheless, the main limitation of **standard VPINNs** is the lack of stability and robustness with respect to the true error, implying poor performance in challenging scenarios [3].

Building on these ideas, **Robust Variational PINNs (RVPINNs)** introduced a residual minimisation framework in **discrete dual norms** [4]. This leads to better-posed formulations with more potent stability properties and well-defined norms for error control. RVPINNs have demonstrated improved robustness and accuracy in approximating **linear elliptic PDEs**. However, the current framework **has not yet been extended to nonlinear or time-dependent partial differential equations (PDEs**), which are prevalent in real-world applications such as fluid flow, nonlinear diffusion, and reaction–transport systems.

This research problem proposal is to address the open research problem of extending the RVPINN methodology to nonlinear and unsteady partial differential equations (PDEs), aiming to develop a general and robust neural approximation framework capable of handling time dependence, nonlinearity, and multiscale features while maintaining the theoretical stability advantages of variational formulations.

[1] Raissi, M., Perdikaris, P., & Karniadakis, G. E. (2019). Physics-informed neural networks: A deep learning framework for solving forward and inverse problems involving nonlinear partial differential equations. *Journal of Computational physics*, *378*, 686-707.

[2] Kharazmi, E., Zhang, Z., & Karniadakis, G. E. (2019). Variational physics-informed neural networks for solving partial differential equations. *arXiv preprint arXiv:1912.00873*.

[3] Berrone, S., Canuto, C., & Pintore, M. (2022). Variational physics informed neural networks: the role of quadratures and test functions. *Journal of Scientific Computing*, *92*(3), 100.

[4] Rojas, S., Maczuga, P., Muñoz-Matute, J., Pardo, D., & Paszyński, M. (2024). Robust variational physics-informed neural networks. *Computer Methods in Applied Mechanics and Engineering*, *425*, 116904.

Project aims

The primary objective of this PhD project is to advance the Robust Variational Physics-Informed Neural Network (RVPINN) framework for the approximation of nonlinear and time-dependent PDEs. The specific aims are:

1. **Formulate and extend the RVPINN framework to nonlinear and time-dependent PDEs**, incorporating appropriate variational principles and discretisation strategies for parabolic and hyperbolic problems.

2. **Analyse the stability and convergence of the extended RVPINN formulations**, with a focus on identifying suitable residual norms, regularisation techniques, and theoretical guarantees under minimal assumptions.

3. **Validate the proposed methods through implementation and benchmarking** on representative nonlinear and unsteady PDEs, comparing performance against standard PINNs and classical numerical solvers.

How skills/experience of the IITB and the Monash supervisor(s) support the proposed project

Highlight the purpose of the collaboration and/or the complementary skills/experience that you bring to the project. Do you have any joint or independent publications in the area of the proposed project?

Myself, Prof. Neela Nataraj from IIT Bombay, have already graduated two students in the IIT Bombay Monash Research Academy and am currently supervising two students; out of which one is in the final year of the Ph.D. programme. The past success stories with Prof. Droniou and the current supervision with Prof. Ricardo inspire me to select a student under the joint PhD programme to work on the current important topic of Neural Network approximation. Dr. Rojas specialises in residual minimisation methods and neural network-based solvers for PDEs. While we have not yet published together, this project presents a timely opportunity for joint research outcomes. I am currently studying literature on Deep Neural Networks, and I think the complementary expertise of Dr. Rojas will help in the scientific advancement of the topic.

What is expected of the student when at IITB and when at Monash?

Highlight how the project will gain from the students stay at IITB and at Monash

At **IIT Bombay**, the student will complete coursework in numerical analysis, PDEs and functional analysis, and begin preliminary research on neural network models for PDEs. This phase will focus on building strong theoretical foundations.

At **Monash University**, the student will develop and implement advanced RVPINN techniques for nonlinear and time-dependent PDEs under the supervision of Dr. Sergio Rojas, in close collaboration with Prof. Neela Nataraj through regular weekly Zoom meetings. The student will also perform computational validation of the methods and contribute to high-quality research publications.

Expected outcomes

Highlight the expected outcomes of the project

- Development of novel and robust neural network-based schemes for solving evolution equations, with a focus on time-dependent and nonlinear PDEs.
- Implementation of reproducible and open-access computational tools to support the broader research community.
- Comprehensive research training for the student in applied mathematics, scientific computing, and machine learning techniques for PDEs.
- Publication of research findings in high-quality, peer-reviewed journals in mathematics and computational science.

How will the project address the Goals of the above Themes?

Describe how the project will address the goals of one or more of the 6 Themes listed above.

The development of robust neural network methods for solving nonlinear and time-dependent PDEs directly contributes to the advancement of **Artificial Intelligence and Advanced Computational Modelling**. The project aligns with this theme by addressing theoretical and practical challenges in AI-based solvers for PDEs, enhancing their reliability, interpretability, and applicability to real-world problems in physics and engineering.

The expected outcomes, such as the formulation of new RVPINN-based schemes, their rigorous analysis, and the creation of open-access computational tools, support the broader goals of enabling high-impact applications in simulation, modelling, and scientific computing. The interdisciplinary nature of the project is also expected to foster collaboration between mathematical modelling, AI, and computational sciences.

How well the IITB and the Monash supervisor(s) know each other

Provide details of previous collaborations (if any). For new collaborators, have you had a chance to meet each other in person or through VC or Skype?

They met in person in May 2025, during a visit of Prof. Neela Nataraj at the School of Mathematics of Monash University.

Potential RPCs from IITB and Monash

Provide names of the potential research progress committee members (RPCs) and describe why they are most suited for the proposed project

- **Prof. Ricardo Ruiz-Baier** (Monash University): An expert in numerical analysis, specialising in finite element methods for partial differential equations, particularly in mixed and multiphysics formulations. His expertise could provide valuable support for the theoretical and analytical components of the project, especially in the development and analysis of variational methods for complex unsteady PDEs.

- **Prof. Santiago Badia** (Monash University): Expert in computational science and scientific computing. He is the principal developer of the Gridap library in Julia for finite element analysis and has conducted significant work on neural network approximation of PDEs. His computational perspective could greatly benefit the implementation and benchmarking components of the project.

- **Prof. Manas Racch** (IIT Bombay): An applied mathematician who has recently joined IIT Bombay, and has expertise in Integral equation methods for the solution of PDEs of mathematical physics, Fast algorithms for applications in electrostatics, acoustics, elasticity, viscous flow, electromagnetics, computational biology, and biomedical imaging. He is also interested in Deep Neural Networks for PDE.

- **Prof. Harsha Hutridurga** (IIT Bombay): An applied mathematician and an Associate Professor in IIT Bombay with vast expertise on homogenization and kinetic theory. He is associated with the IIT Bombay Monash Research Academy and is currently guiding a student. He is also interested in Deep Neural Networks for PDE.

Capabilities and Degrees Required

List the ideal set of capabilities that a student should have for this project. Feel free to be as specific or as general as you like. These capabilities will be input into the online application form and students who opt for this project will be required to show that they can demonstrate these capabilities.

- Master in Mathematics (or equivalent), with focus on numerical analysis.
- Basic theory of Partial Differential Equations.
- Strengths in Linear Algebra and Functional Analysis.
- Knowledge of classical Finite Element Methods or Neural Networks.
- Good coding skills and knowledge of programming languages, with capacity to learn new ones if not already mastered (MATLAB, Python, Julia, etc.)

Necessary Courses

Name three tentative courses relevant to the project that the student should complete during his/her coursework at IITB (the student will require to secure 8 point in these courses)

Numerical Analysis
Partial Differential Equations
Functional Analysis

Potential Collaborators

Please visit the IITB website www.iitb.ac.in OR Monash Website www.monash.edu to highlight some potential collaborators that would be best suited for the area of research you are intending to float.

Prof. Ricardo Ruiz-Baier, Monash University
Prof. Santiago Badia, Monash University
Prof. Shiva Gopalakrishnan, IIT Bombay
Prof. Harsha Hutridurga, IIT Bombay
Prof. Manas Racch, IIT Bombay
Prof. Debanjana Mitra, IIT Bombay

Keywords relating to this project to make it easier for the students to apply.

Numerical Methods for Partial Differential Equations
Physics-Informed Neural Networks
Finite Elements
Computer Simulation

Maths
Computational Fluid Dynamics and Mechanics